Measuring Communication in Automata Systems

Martin Kutrib Andreas Malcher

Institut für Informatik, Universität Giessen, Germany

Systems of Interacting Finite Automata

- Weakly parallel models: constant number of synchronous and autonomous finite automata.
- Massively parallel models: arrays of homogeneously connected copies of finite automata.

Typical Problems and Questions

- How much communication is necessary for a computation?
- How can the cooperation of the cells be organized optimally?
- From the viewpoint of energy and the costs of communication links, it would be desirable to communicate a minimal number of times with a minimal bandwidth of the links.

Measuring the Communication

- Quantitatively: the number of messages allowed to be sent by the components.
- Qualitatively: the bandwidth of the communication links between the components, that is, the number of different messages available.

Overview

- Weakly Parallel Systems
  - Broadcasting Messages
  - Requesting Messages

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- Weakly Parallel Systems
  - Broadcasting Messages
  - Requesting Messages
- Massively Parallel Systems
  - Limiting the Inter-Cell Bandwidth
  - Limiting the Number of Messages
  - Both Restrictions At Once

Weakly Parallel Systems

Broadcasting Messages

- Several technical aspects have to be considered.
- Is it allowed that more than one component broadcast messages at the same time?

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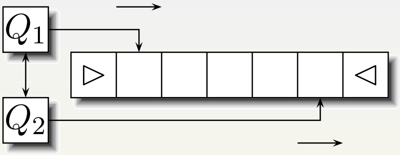
- Is it allowed that more than one component broadcast messages at the same time?
- If yes, what happens at the recipients?
- Is only one message processed?
- If yes, which one?
- Or else are all messages processed?
- If yes, in which order or in parallel?

Weakly Parallel Systems – Broadcasting Messages

Starting Point

Deterministic one-way two-party finite automata systems.



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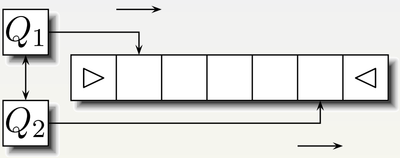

Set of Messages: B

Broadcast functions:

$$\mu_i : Q_i \times (\Sigma \cup \{\triangleright, \triangleleft\}) \to B \cup \{\bot\},$$

where $\bot \notin B$ means nothing to send

Transition functions:

$$\delta_i : Q_i \times (\Sigma \cup \{\triangleright, \triangleleft\}) \times (B \cup \{\bot\}) \to Q_i \times \{0, 1\}$$

Weakly Parallel Systems – Broadcasting Messages

Language acceptance

- Whenever the transition function of (at least) one of the components is undefined the whole systems halts.
- The input is accepted if at least one of the components is in an accepting state at this moment.

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- For a mapping $f : \mathbb{N} \to \mathbb{N}$, a finite automata system is said to be communication bounded by f, if all accepted inputs w are accepted with computations where the total number of messages sent is bounded by $f(|w|)$.

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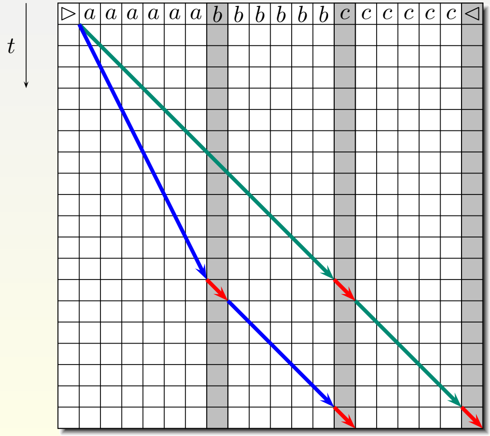
- For a mapping $f : \mathbb{N} \to \mathbb{N}$, a finite automata system is said to be communication bounded by f, if all accepted inputs w are accepted with computations where the total number of messages sent is bounded by $f(|w|)$.

- We denote the class of $\mathrm{DFAS}(2)$ that are communication bounded by $O(f)$ by (f)-$\mathrm{DFAS}(2)$.

Weakly Parallel Systems – Broadcasting Messages

Example

The language $\{ a^n b^n \mid n \geq 1 \}$ is accepted by some (1)-$\mathrm{DFAS}(2)$.
The language $\{ a^n b^n c^n \mid n \geq 1 \}$ is accepted by some (2)-$\mathrm{DFAS}(2)$.



Weakly Parallel Systems – Broadcasting Messages

Problem

How much communication is necessary to accept a non-semilinear language?

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Upper bound:

$$L_{expo} = \{ a^{2^0}ba^{2^1}b\cdots ba^{2^m} \mid m \geq 1 \}$$

is accepted by a $(\log(n))$-DFAS(2).

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Lower bound:
Any $(const)$-$\mathrm{DFAS}(2)$ can effectively be simulated by a deterministic (r, s)-reversal bounded two-way k-counter machine, where k, r, and s are constants.

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Theorem **[Ibarra 1978]**

Any language accepted by a deterministic reversal-bounded two-way finite-counter machine is semilinear.

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Lower bound: $\omega(1)$ communications.

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Theorem **[Jurdziński, Kutyłowski 2001]**

There is no language accepted by some $\mathrm{DFAS}(k)$ that requires more than constant, that is $\omega(1)$, and less than logarithmic, that is $o(\log(n))$, messages to be sent.

Weakly Parallel Systems – Broadcasting Messages

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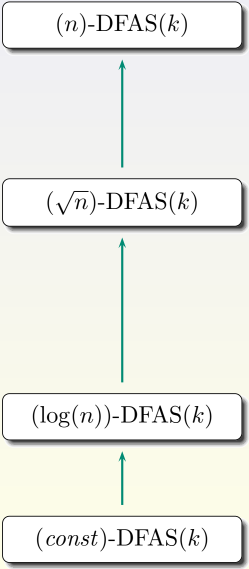

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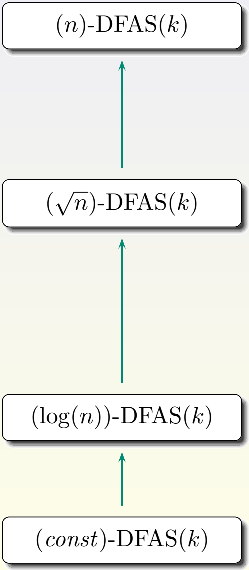
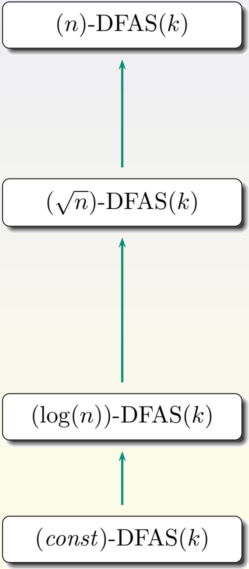


Open problem:
Is there a hierarchy dependent on the constant?

Weakly Parallel Systems – Broadcasting Messages



Witness language:

$$\{\, x_1 a^1 x_2 \cdots x_m a^{2m-1} c x_1 a^{2m+1} x_2 \cdots x_m a^{4m-1} \mid m \geq 1 \text{ and } x_i \in \{0,1\}, 1 \leq i \leq m \,\}$$

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Weakly Parallel Systems – Broadcasting Messages

(n)-DFAS(k)

↑

Witness language:

$\{ wcw \mid w \in \{0,1\}^* \}$

$(\sqrt{n})$-DFAS(k)

↑

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$(\log(n))$-DFAS(k)

↑

$(const)$-DFAS(k)

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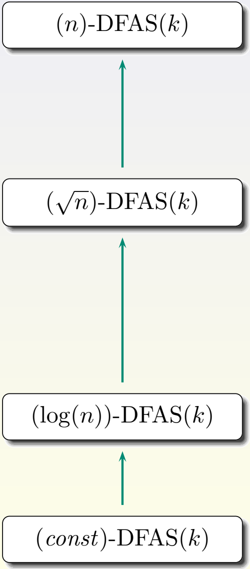

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Is there an infinite hierarchy in between $O(n)$ and $O(\log(n))$?

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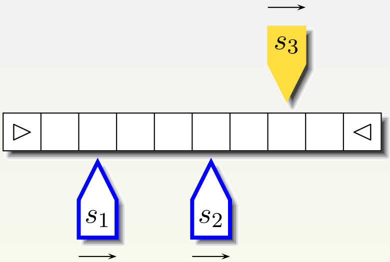
Weakly Parallel Systems

Requesting Messages

Deterministic parallel communicating finite automata system of degree k ($\mathrm{DPCFA}(k)$).



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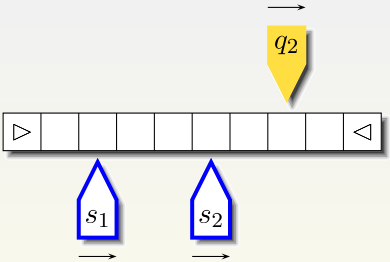

Transition functions:

$$\delta_i : Q_i \times (\Sigma \cup \{\rhd, \lhd\}) \to (Q_i \cup \{q_1, q_2, \ldots, q_k\}) \times \{0, 1\}$$

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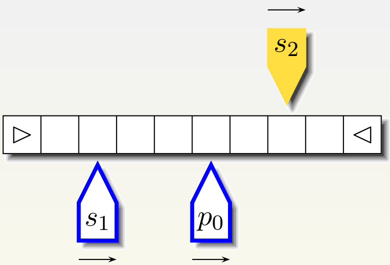

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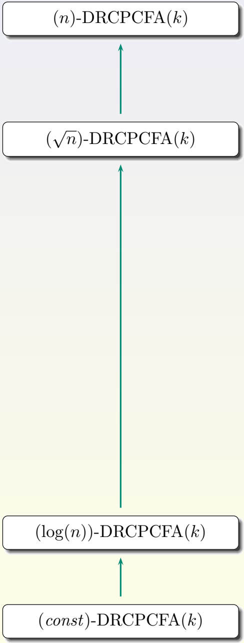
Here: Returning Centralized Mode

Weakly Parallel Systems – Requesting Messages



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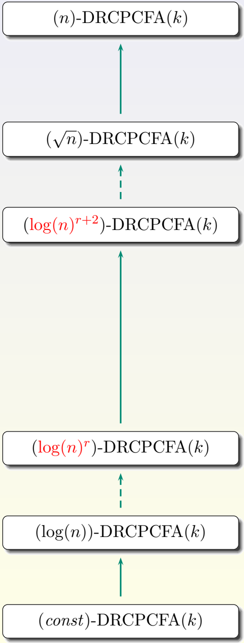

Open problem:
Is there an infinite hierarchy in between $O(n)$ and $O(\sqrt{n})$?

Witness language:

$$L_r = \{\, \$_1 x_1 x_2 \cdots x_\ell \$_2 w_1' w_2' \cdots w_m' w_{m+1} \cdots w_\ell \$_3 \; w_1' w_2' \cdots w_m' w_{m+1} \cdots w_\ell \$_4 a^{2^0} bba^{2^1} bb \cdots a^{2^{m-1}} bb\& \mid m \geq 1, x_1 x_2 \cdots x_\ell \text{ is the valid computation of } M_r \text{ on input } a^m,\, w_i' \in \{0', 1'\}, 1 \leq i \leq m,\, w_i \in \{0, 1\},\, m+1 \leq i \leq \ell \,\}$$

Open problem:
Is there a hierarchy dependent on the constant?

Weakly Parallel Systems

Decidability

Emptiness, finiteness, infiniteness, universality, inclusion, equivalence, regularity, and context-freeness are not semidecidable for $(\log(n))\text{-DFAS}(2)$ and $(\log(n))\text{-DRCPCFA}(4)$.

Weakly Parallel Systems

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Emptiness, finiteness, infiniteness, universality, inclusion, equivalence, regularity, and context-freeness are
not semidecidable
for $(\log(n))$-DFAS(2) and $(\log(n))$-DRCPCFA(4).

Emptiness, finiteness, infiniteness, universality, inclusion, equivalence are
decidable
for $(const)$-DFAS(k) and $(const)$-DRCPCFA(k).

Weakly Parallel Systems

Decidability or Computability of Communication Bounds

Problem
Decide for a given $\mathrm{DRCPCFA}(k)$ M and a given function f whether or not M is communication bounded by f.

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Decide for a given $\mathrm{DRCPCFA}(k)$ M and a given function f whether or not M is communication bounded by f.

Let $k \geq 3$ be an integer, $f \in o(n)$, and M be a $\mathrm{DRCPCFA}(k)$. Then it is not semi-decidable whether M is communication bounded by f.

Weakly Parallel Systems

Decidability or Computability of Communication Bounds

Problem
Decide for a given $\mathrm{DRCPCFA}(k)$ M whether or not all accepting computations of M are communication bounded by some function $f \in O(1)$ and, if it is, to compute the precise constant.

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Decidability or Computability of Communication Bounds

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Decide for a given $\mathrm{DRCPCFA}(k)$ M whether or not $L(M)$ is accepted by some $\mathrm{DRCPCFA}(k)$ whose accepting computations are communication bounded by some function $f \in O(1)$.

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Weakly Parallel Systems

Untouched Problems

Communication in deterministic non-centralized or non-returning $\mathrm{DRCPCFA}(k)$.

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Open Problems

Communication hierarchies within the constants and in between $O(n)$ and $O(\sqrt{n})$.

Weakly Parallel Systems

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Communication in deterministic non-centralized or non-returning $\mathrm{DRCPCFA}(k)$.

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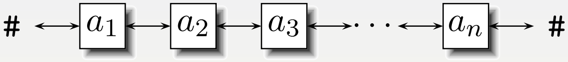
Untouched Problems

Descriptional complexity of communication.

Massively Parallel Systems

Two-way cellular automata (CA)



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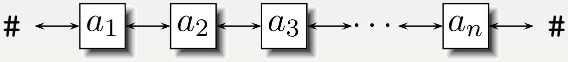


Set of Messages: B

Communication functions:

$$b_l, b_r : (S \cup \{\#\}) \to (B \cup \{\perp\}),$$

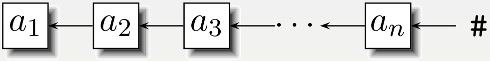
where $\perp \notin B$ means nothing to send

Transition function:

$$\delta : (B \cup \{\perp\}) \times S \times (B \cup \{\perp\}) \to S$$

Massively Parallel Systems

One-way cellular automata (OCA)



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Massively Parallel Systems

Language acceptance

- An input string (word) is accepted, if the leftmost cell enters an accepting state during the course of its computation.

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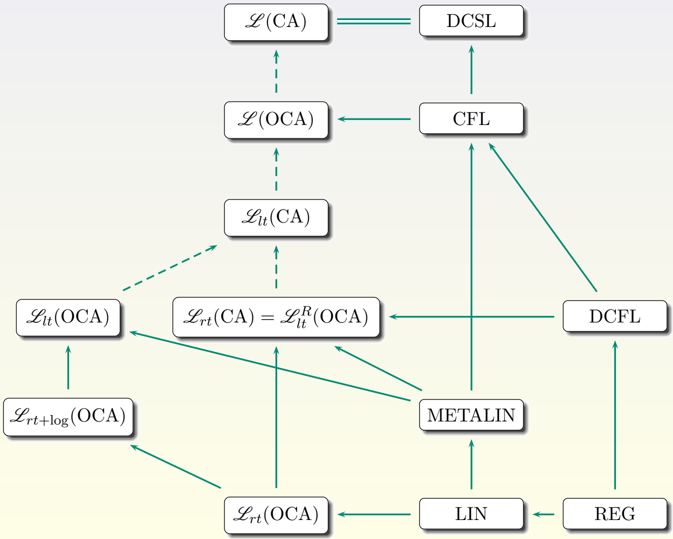
- If all $w \in L(M)$ are accepted with at most $|w|$ time steps, then M is a real-time CA.
- The corresponding family of languages is denoted by $\mathscr{L}_{rt}(\mathrm{CA})$.

Massively Parallel Systems

Cellular automata with unlimited communication



Massively Parallel Systems

What Devices with Limited Inter-Cell Bandwidth Can Do

- There is a time-optimal one-message solution of the famous Firing Squad Synchronization Problem [Mazoyer 1989].

- There are rather complex unary languages accepted by one-message CA in real time [Umeo, Kamikawa 2002,2003]:

  - $\{ a^{2^n} \mid n \geq 1 \} \in \mathscr{L}_{rt}(\mathrm{CA}_1)$
  - $\{ a^{n^2} \mid n \geq 1 \} \in \mathscr{L}_{rt}(\mathrm{CA}_1)$
  - $\{ a^p \mid p \text{ is prim} \} \in \mathscr{L}_{rt}(\mathrm{CA}_1)$
  - $\{ a^p \mid p \text{ is a Fibonacci number} \} \in \mathscr{L}_{rt}(\mathrm{CA}_1)$

Massively Parallel Systems

What Devices with Limited Inter-Cell Bandwidth Cannot Do

Observation: $\mathrm{REG} \not\subseteq \mathscr{L}_{rt}(\mathrm{CA}_k)$, for all $k \geq 1$.

Massively Parallel Systems

What Devices with Limited Inter-Cell Bandwidth Cannot Do

Observation: $\mathrm{REG} \not\subseteq \mathscr{L}_{rt}(\mathrm{CA}_k)$, for all $k \geq 1$.

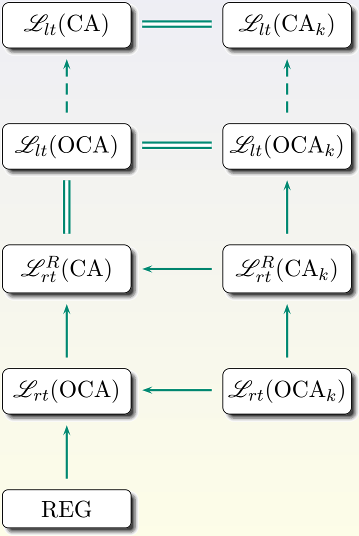
$$\mathscr{L}_{rt}(\mathrm{CA}_k) \subset \mathscr{L}_{rt}(\mathrm{CA}_{k+1})$$

and

$$\mathscr{L}_{rt}(\mathrm{OCA}_k) \subset \mathscr{L}_{rt}(\mathrm{OCA}_{k+1}),$$

for all $k \geq 1$.

Massively Parallel Systems



Massively Parallel Systems

Decidability

Emptiness, universality, finiteness, infiniteness, equivalence, inclusion, regularity, and context-freeness are
undecidable
for real-time OCA_1.

Limiting the Number of Messages

- Whenever a communication symbol is sent, the corresponding link is used.
- We do not distinguish whether either or both neighboring cells use the link.

Massively Parallel Systems

Limiting the Number of Messages

- Let $f : \mathbb{N} \to \mathbb{N}$ be a mapping.
- If all $w \in L(M)$ are accepted with computations such that any link is used at most $O(f(|w|))$ times, then M is said to be max communication bounded by f.

Massively Parallel Systems

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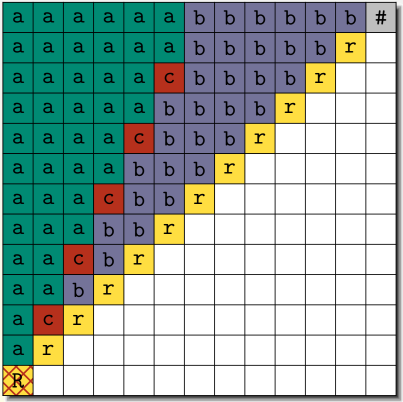
- If all $w \in L(M)$ are accepted with computations such that any link is used at most $O(f(|w|))$ times, then M is said to be max communication bounded by f.
- If all $w \in L(M)$ are accepted with computations where the total number of all communications does not exceed $O(f(|w|))$, then M is said to be sum communication bounded by f.

Limiting the Number of Messages

Examples

➜ $\{\, a^n b^n \mid n \geq 1 \,\} \in \mathscr{L}_{rt}(\mathrm{MC}(const)\text{-OCA})$



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- $\{ a^n b^m c^n d^m \mid n, m \geq 1 \} \in \mathscr{L}_{rt}(\mathrm{MC}(const)\text{-OCA})$

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- $\{\, a^n b^m c^n d^m \mid n, m \geq 1 \,\} \in \mathscr{L}_{rt}(\mathrm{MC}(const)\text{-OCA})$

- $\{\, a^n b^{n+\lfloor\sqrt{n}\rfloor} \mid n \geq 1 \,\} \in \mathscr{L}_{rt}(\mathrm{MC}(const)\text{-OCA})$

Limiting the Number of Messages

Theorem **[Vollmar 1981/1982]**

$\mathscr{L}_{rt}(\mathrm{MC}(const)\text{-CA}) \subset \mathscr{L}_{rt}(\mathrm{SC}(n)\text{-CA})$

$\mathscr{L}_{rt}(\mathrm{MC}(const)\text{-CA}) \subset \mathscr{L}_{rt}(\mathrm{MC}(\sqrt{n})\text{-CA}) \subset \mathscr{L}_{rt}(\mathrm{MC}(n)\text{-CA})$

$\mathscr{L}_{lt}(\mathrm{MC}(const)\text{-CA}) \subset \mathsf{NL}$

Limiting the Number of Messages

Let $f : \mathbb{N} \to \mathbb{N}$ be a function. If $f \in o(n^2/\log(n))$, then

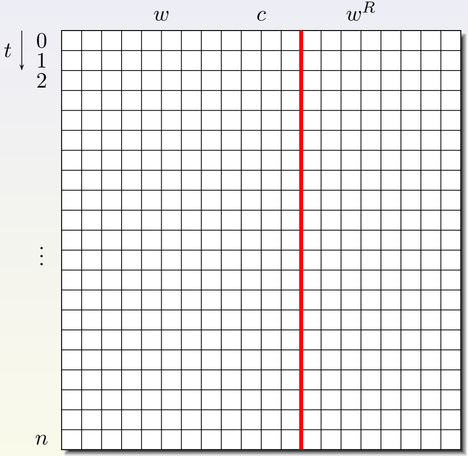
$$\{\, wcw^R \mid w \in \{a,b\}^+ \,\}$$

is not accepted by any real-time $\mathrm{SC}(f)$-CA.

Limiting the Number of Messages

$f \in o(n^2/\log(n))$

$\{\, wcw^R \mid w \in \{a,b\}^+ \,\}$
$\notin \mathscr{L}(\mathrm{SC}(f)\text{-CA})$



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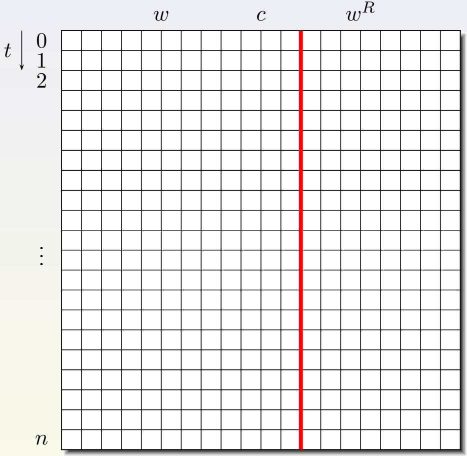

Messages: $r \le |w| / \log(|w|)$

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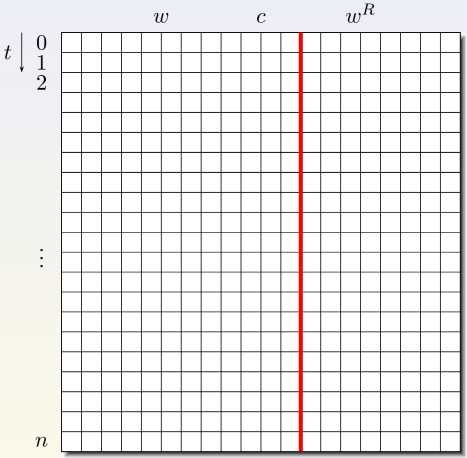

Messages: $r \leq |w|/\log(|w|)$

Possibilities: $(|B|+1)^2 - 1$

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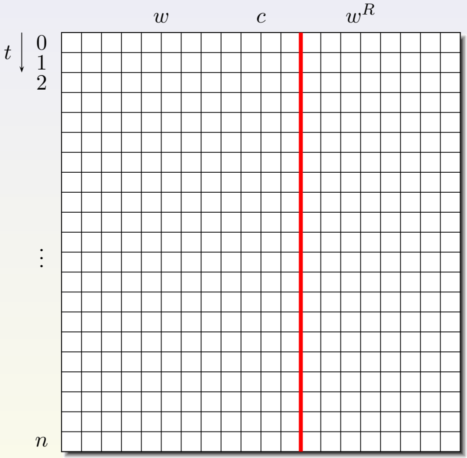
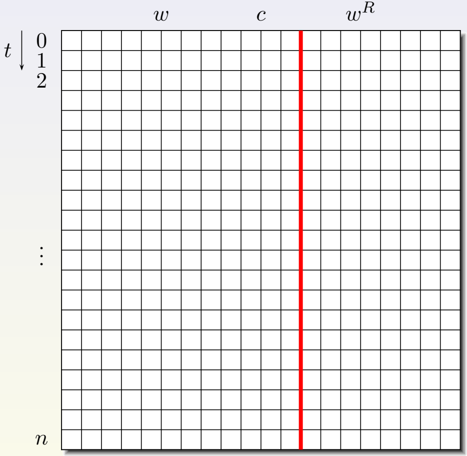
$\notin \mathscr{L}(\mathrm{SC}(f)\text{-CA})$



Messages: $r \leq |w|/\log(|w|)$

Possibilities: $((|B|+1)^2-1)^r$

Limiting the Number of Messages



$f \in o(n^2/\log(n))$

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Possibilities: $\binom{n}{r}((|B|+1)^2-1)^r$

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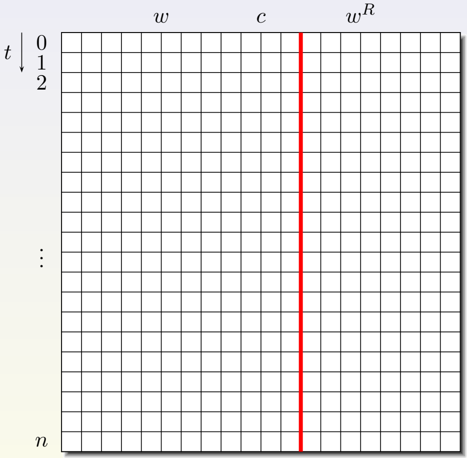

Messages: $r \le |w| / \log(|w|)$

Possibilities: $\binom{n}{r}((|B|+1)^2-1)^r \le 2^{k_0 \log(n) r} \le \sqrt{2^{|w|}}$

Limiting the Number of Messages

Witness languages

$$\varphi_i(n) = \begin{cases} 2^n & \text{if } i = 1 \\ 2^{\varphi_{i-1}(n)} & \text{if } i \geq 2 \end{cases}$$

$$L_i = \{\, w\$^{\varphi_i(|w|)-2|w|} w^R \mid w \in \{a,b\}^+ \,\}$$

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Infinite hierarchy

Let $i \geq 0$ be an integer. Then

$$\mathscr{L}_{rt}(\mathrm{SC}(n \log^{[i+1]}(n))\text{-CA}) \subset \mathscr{L}_{rt}(\mathrm{SC}(n \log^{[i]}(n))\text{-CA}).$$

Limiting the Number of Messages

Decidability

By reduction of Hilbert's tenth problem:

Emptiness, universality, finiteness, infiniteness, equivalence, inclusion, regularity, and context-freeness are
undecidable
for real-time $\mathrm{MC}(const)$-OCA.

Both Restrictions At Once

Emptiness, finiteness, infiniteness, equivalence, inclusion, regularity, and context-freeness are undecidable for real-time $\mathrm{SC}(n)$-OCA_1 and $\mathrm{MC}(const)$-OCA_1.

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Emptiness, finiteness, infiniteness, equivalence, inclusion, regularity, and context-freeness are
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for real-time $\mathrm{SC}(n)$-OCA_2 and $\mathrm{MC}(\log n)$-OCA_2 accepting bounded languages.

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Open Problems

How about real-time $\mathrm{SC}(n)$-OCA_1 or $\mathrm{MC}(o(\log n))$-OCA_1?

Are there other natural restrictions that yield decidable properties?