Deterministic Set Automata

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Finite Automata and Storage Media

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- ➜ Closure under many operations.
- ➜ Decidability of almost all decidability questions and, furthermore, often solvable in polynomial time.
- ➜ Efficient minimization algorithms exist.
- ➜ The computational power is weak since only regular languages are accepted.

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Study deterministic finite automata extended with some storage medium.

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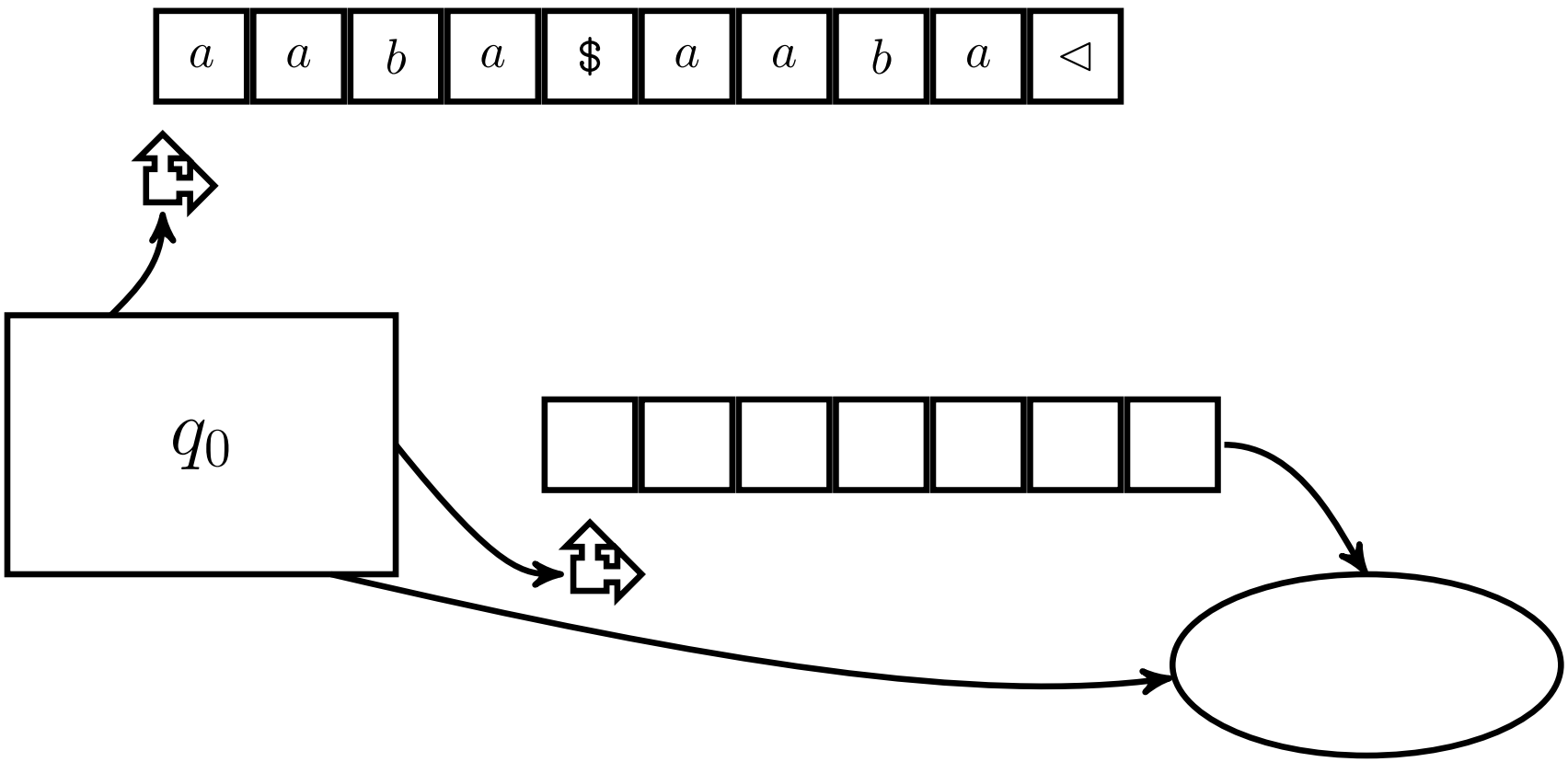
- ➜ In 1995 [Lange, Reinhardt] introduced a model which allows to store and test words in a set.
- ➜ It works nondeterministically, allows no remove operations on the set, and the test operation implicitly adds the word tested to the set.
- ➜ Closure properties of the model, the emptiness problem, and the word problem are studied.

Deterministic Set Automata – Example



Definition

A deterministic set automaton (DSA) is a system $M = \langle S, \Sigma, \Gamma, \lhd, \delta, s_0, F \rangle$, where

- S is the finite set of internal states,
- Σ is the finite set of input symbols,
- Γ is the finite set of tape symbols,
- $\lhd \notin \Sigma$ is the right endmarker,
- $s_0 \in S$ is the initial state,
- $F \subseteq S$ is the set of accepting states, and
- δ is the partial transition function where:

$$\delta : S \times (\Sigma \cup \{\lambda, \lhd\}) \to (S \times (\Gamma^* \cup \{\mathtt{in}, \mathtt{out}\})) \cup (S \times \{\mathtt{test}\} \times S)$$

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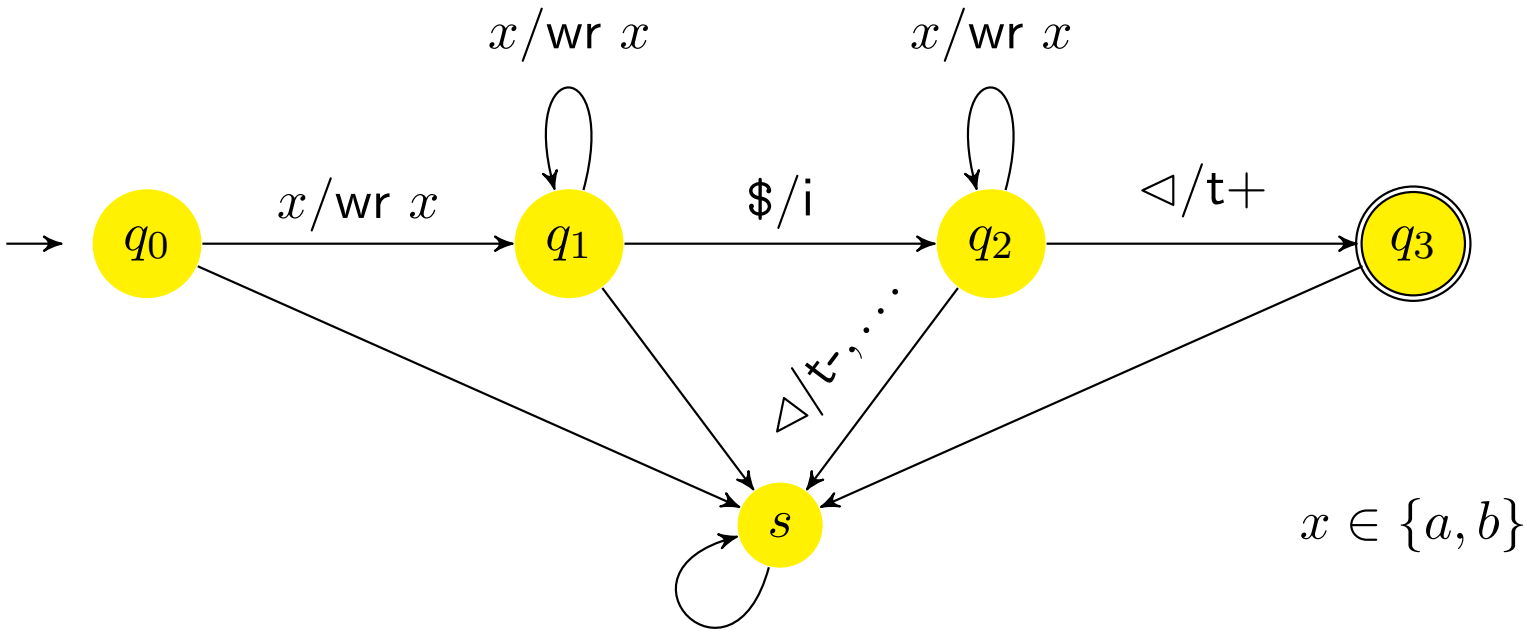

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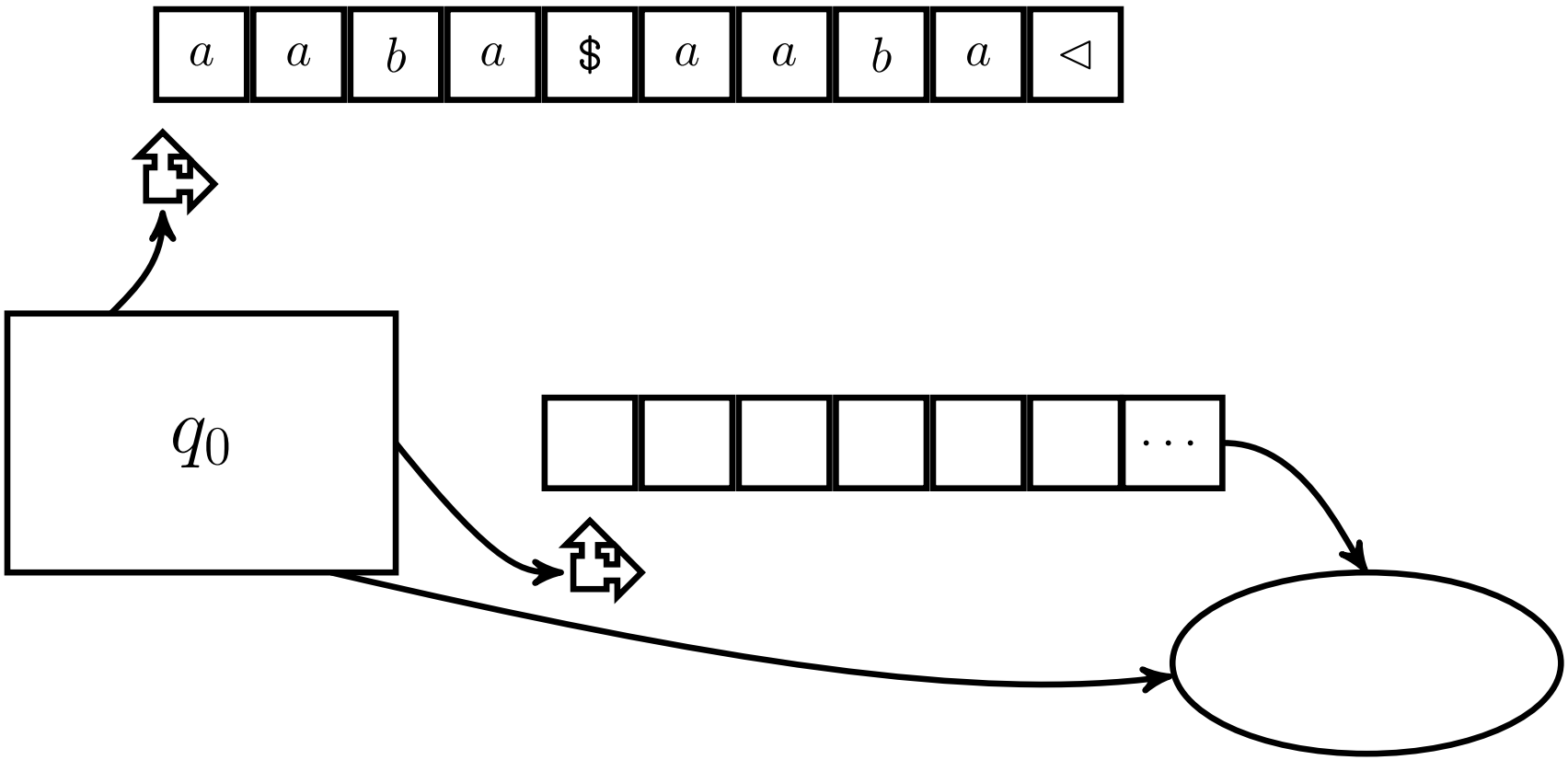

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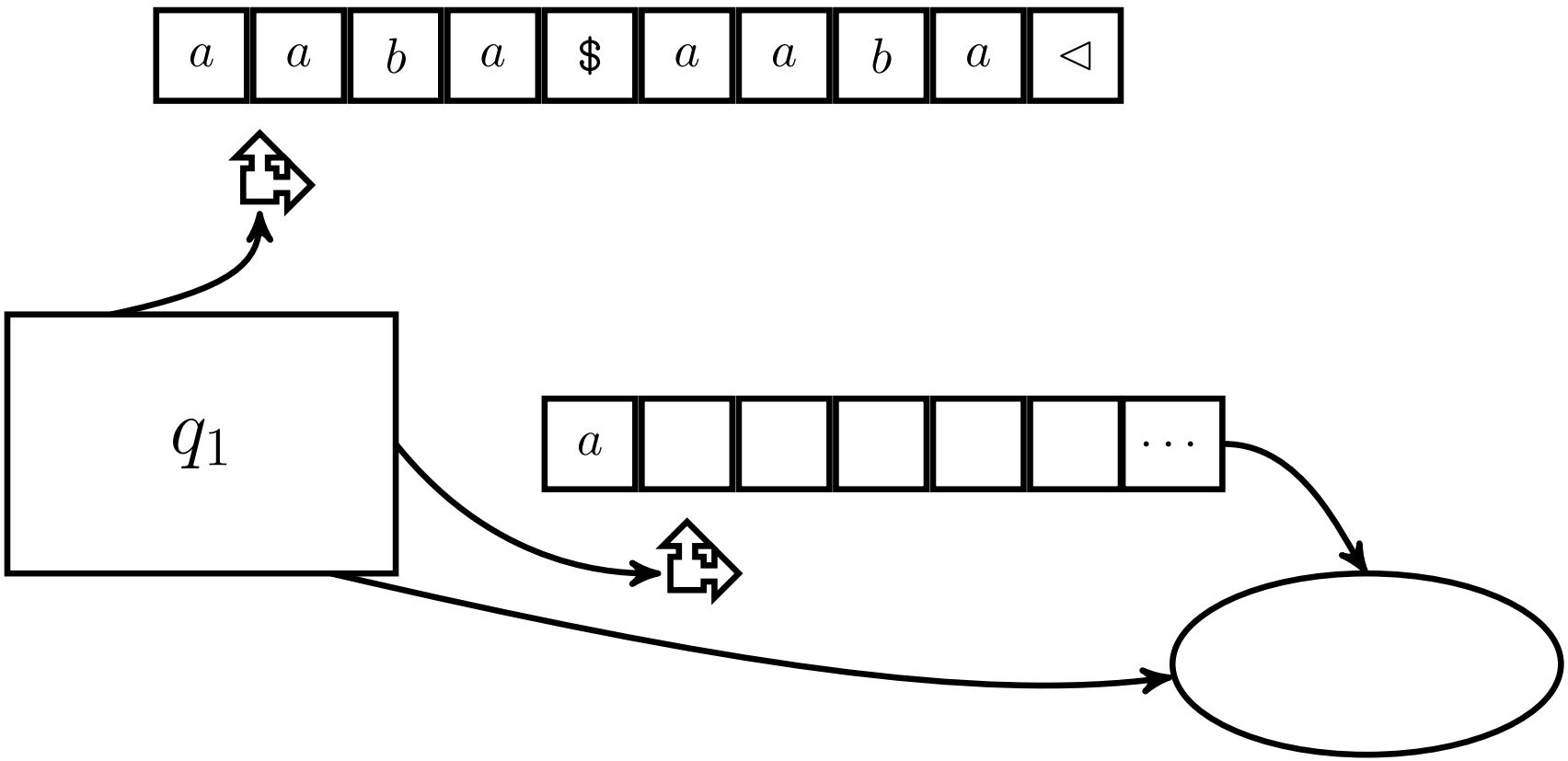

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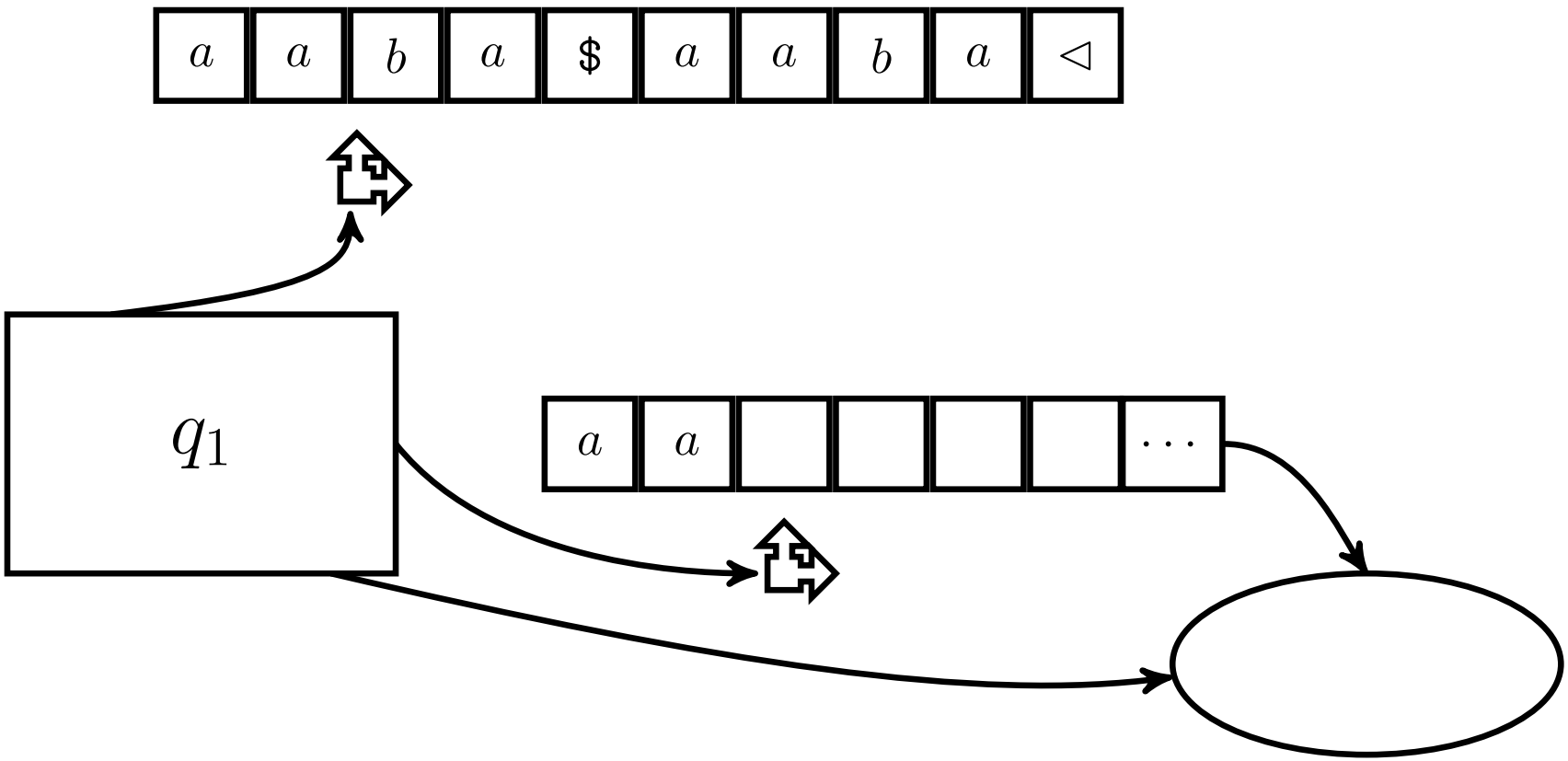

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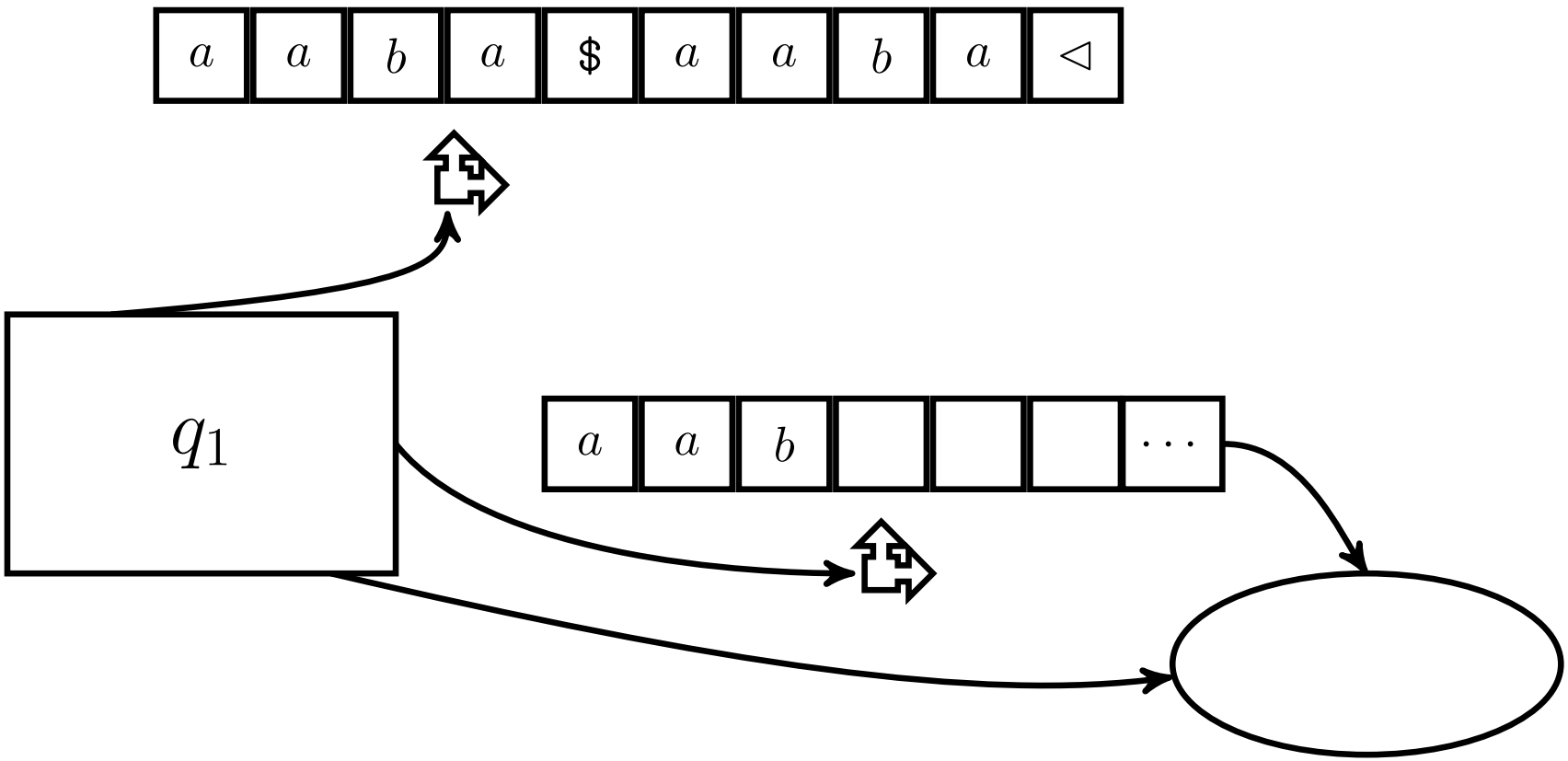

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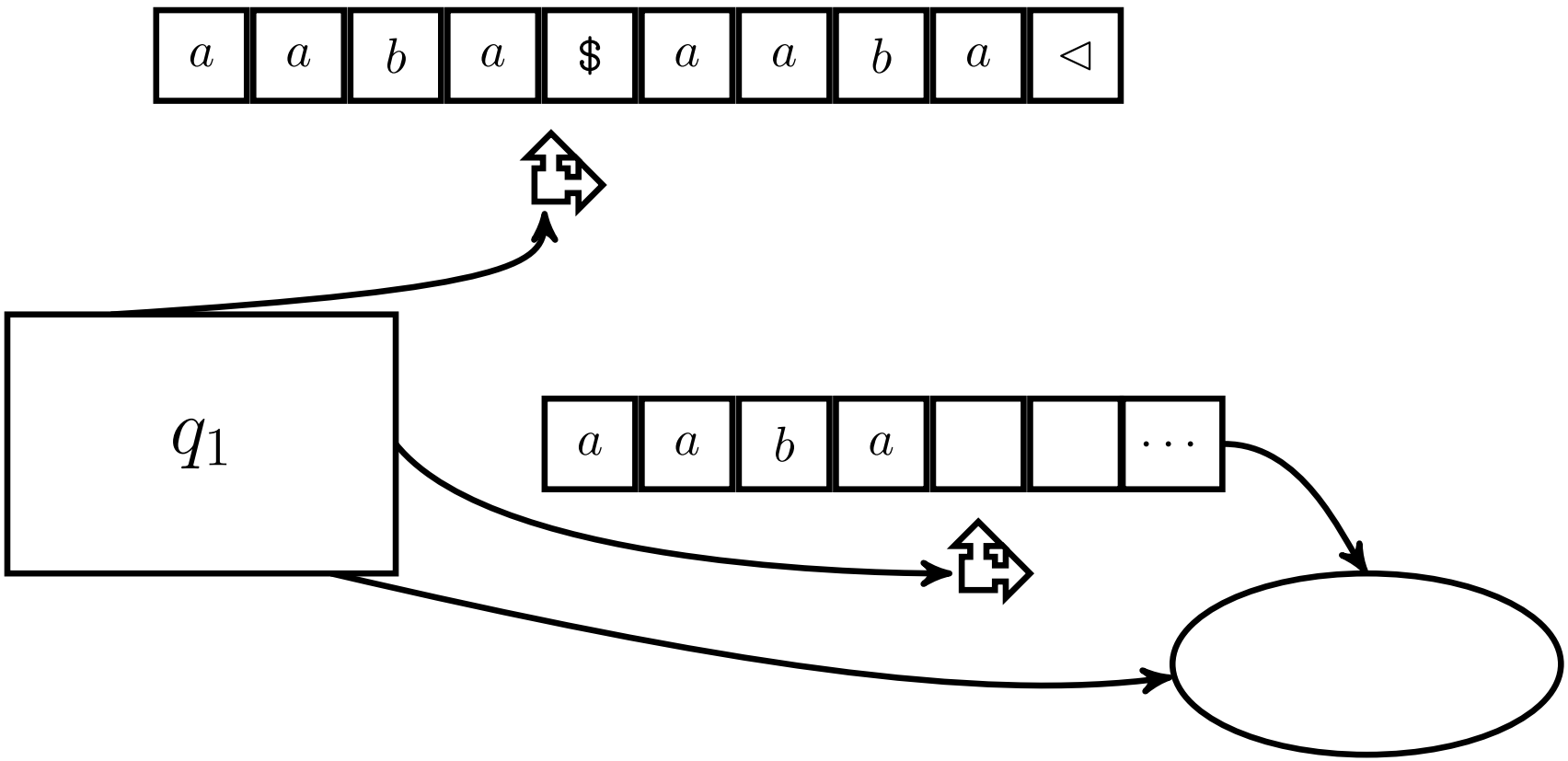

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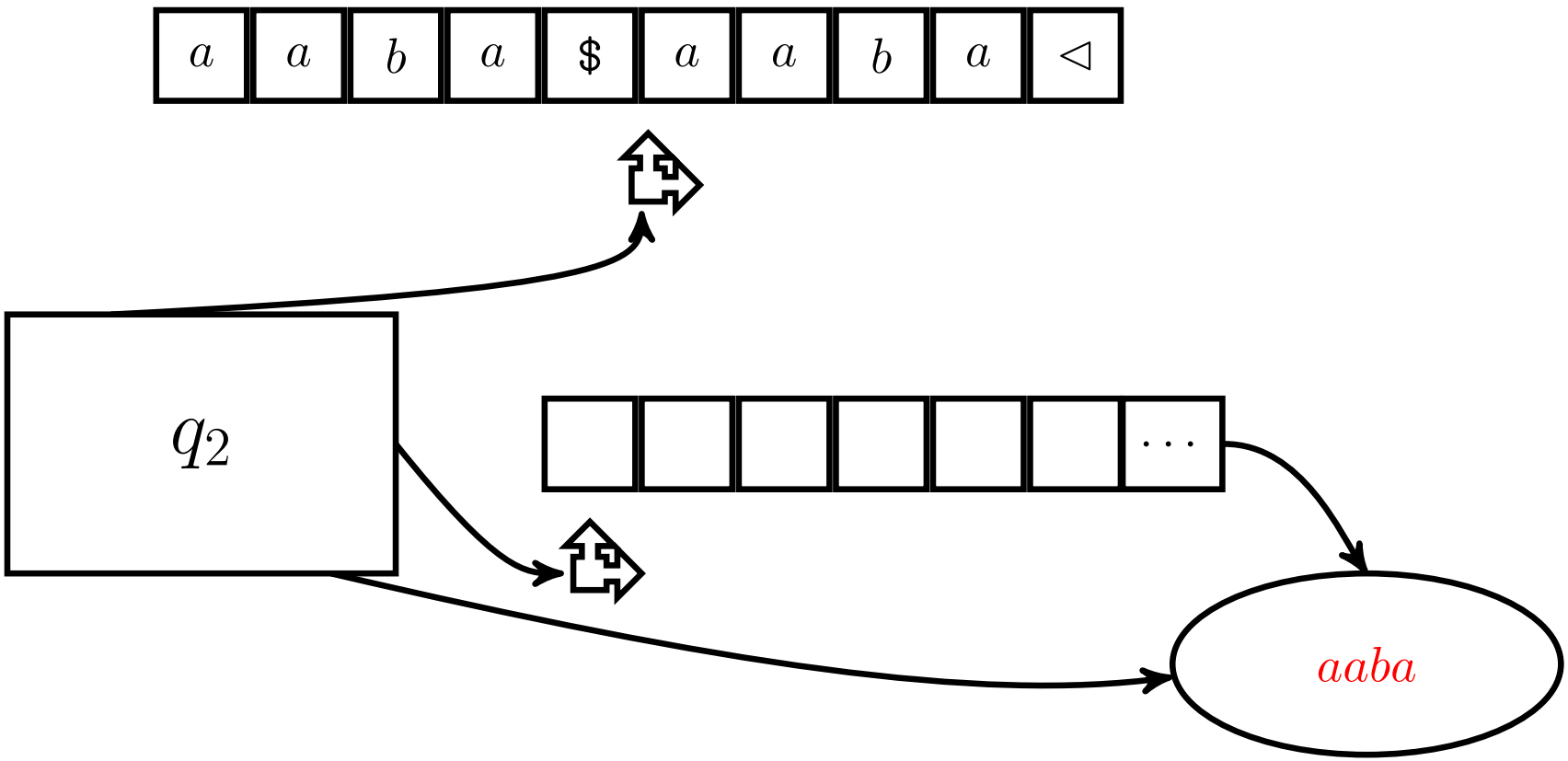

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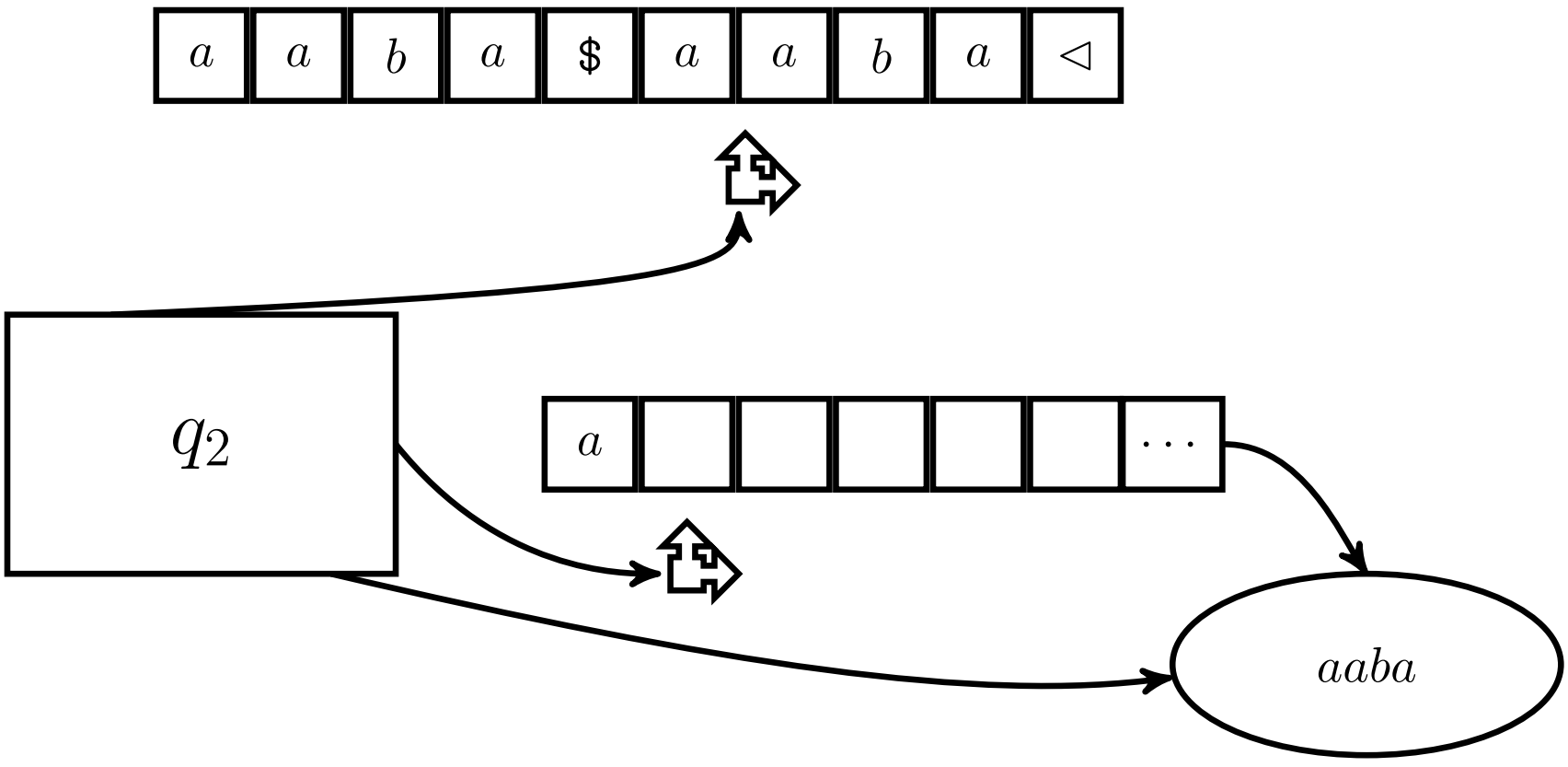

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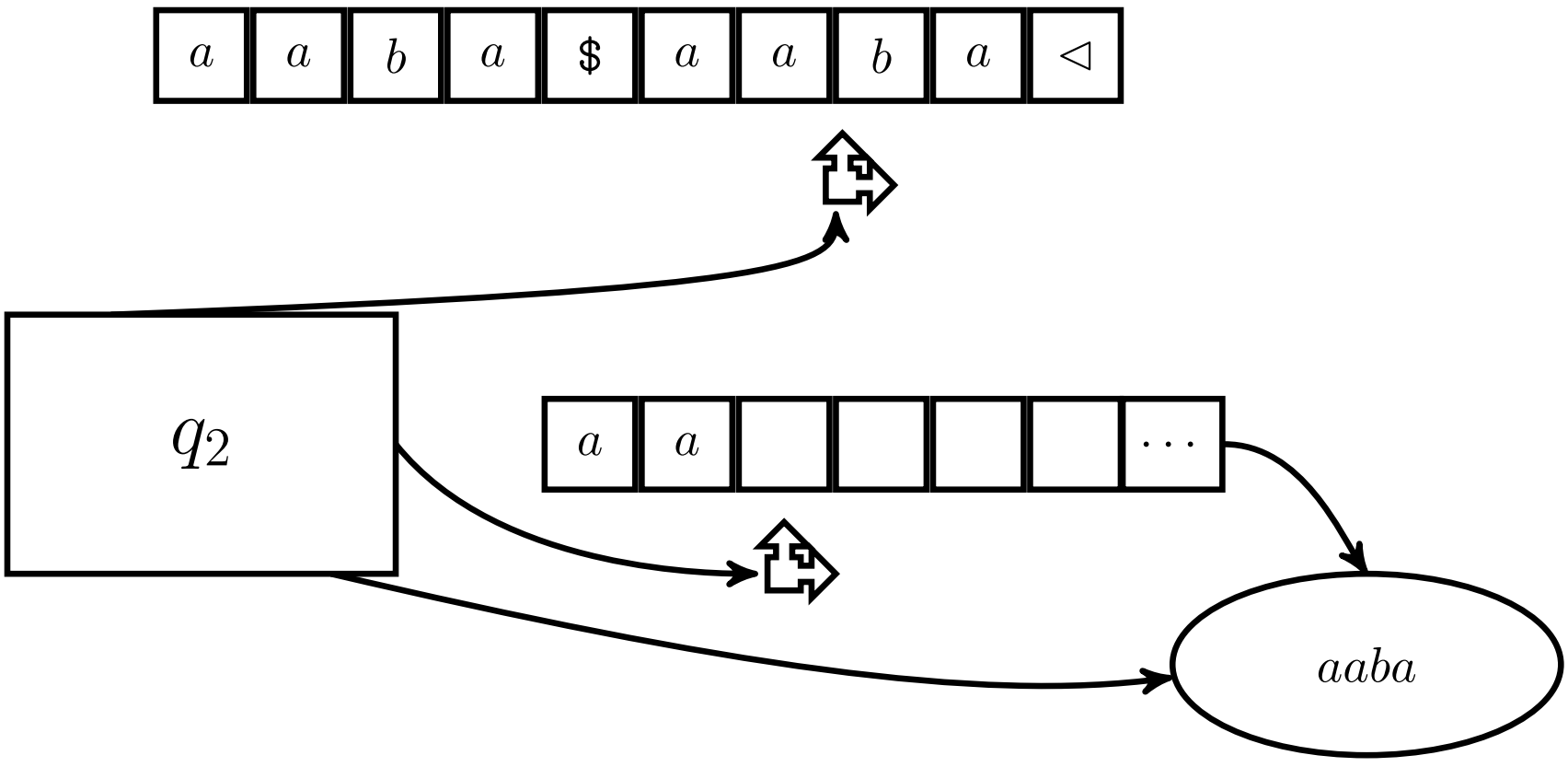

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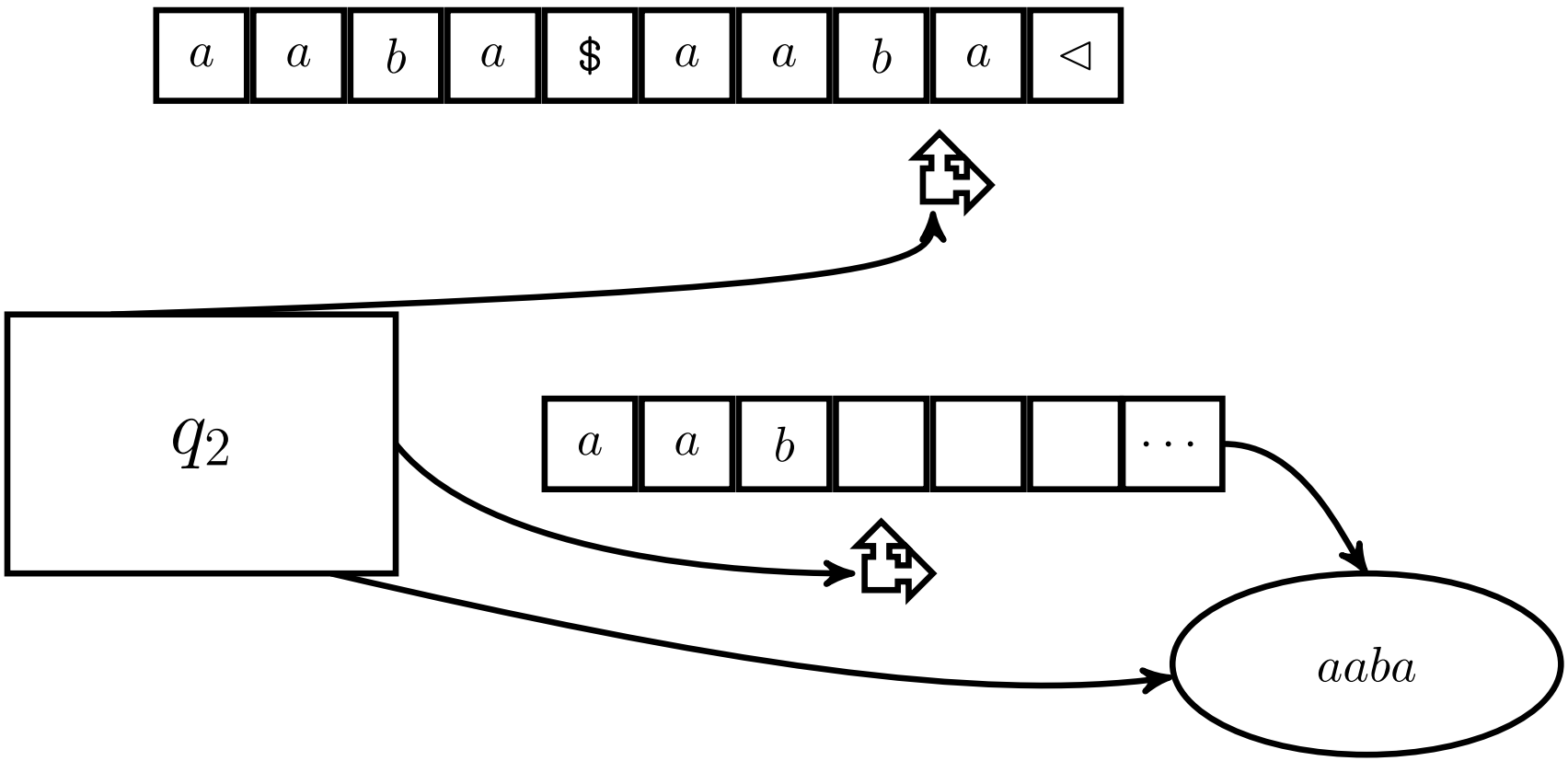

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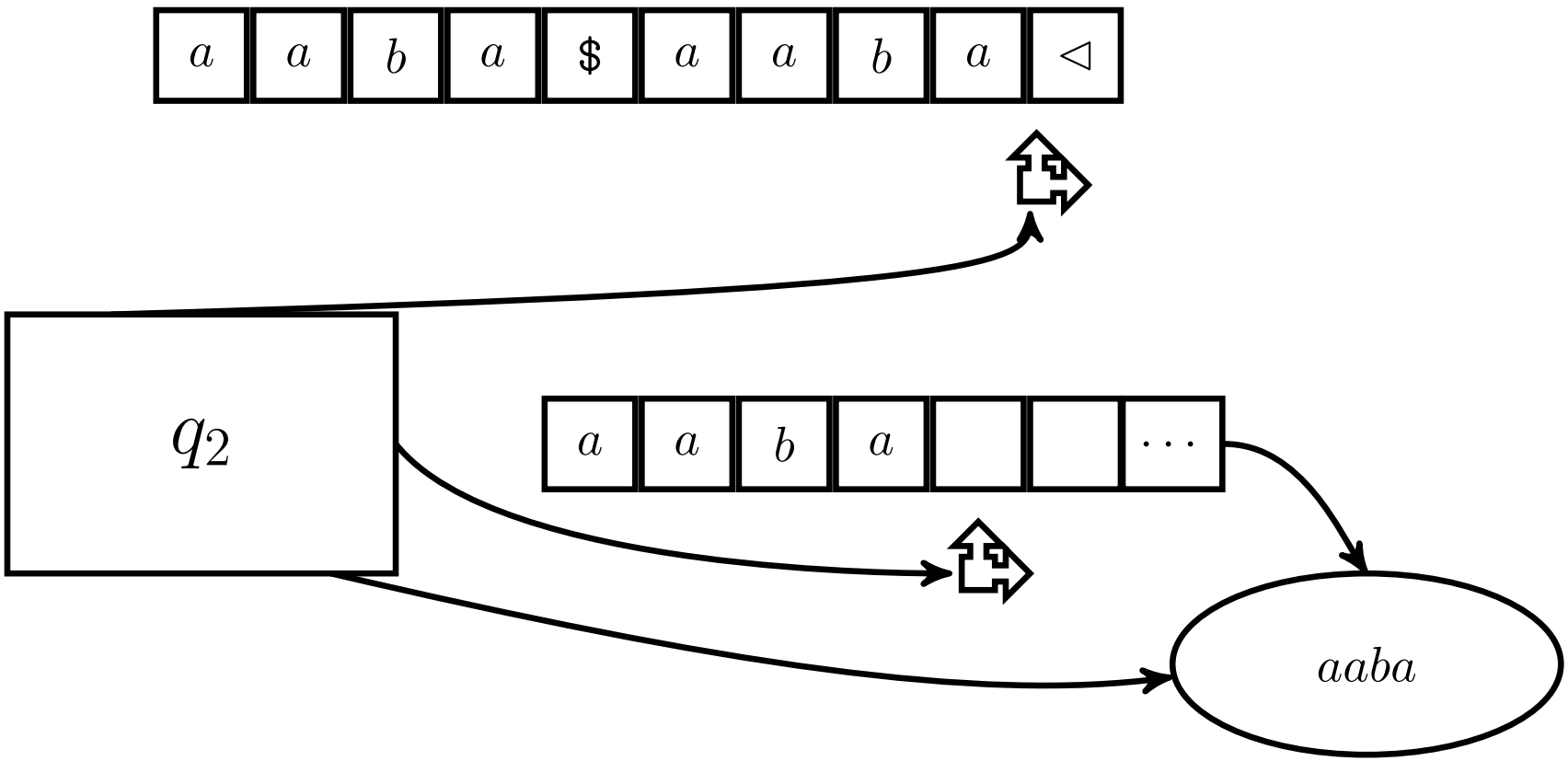

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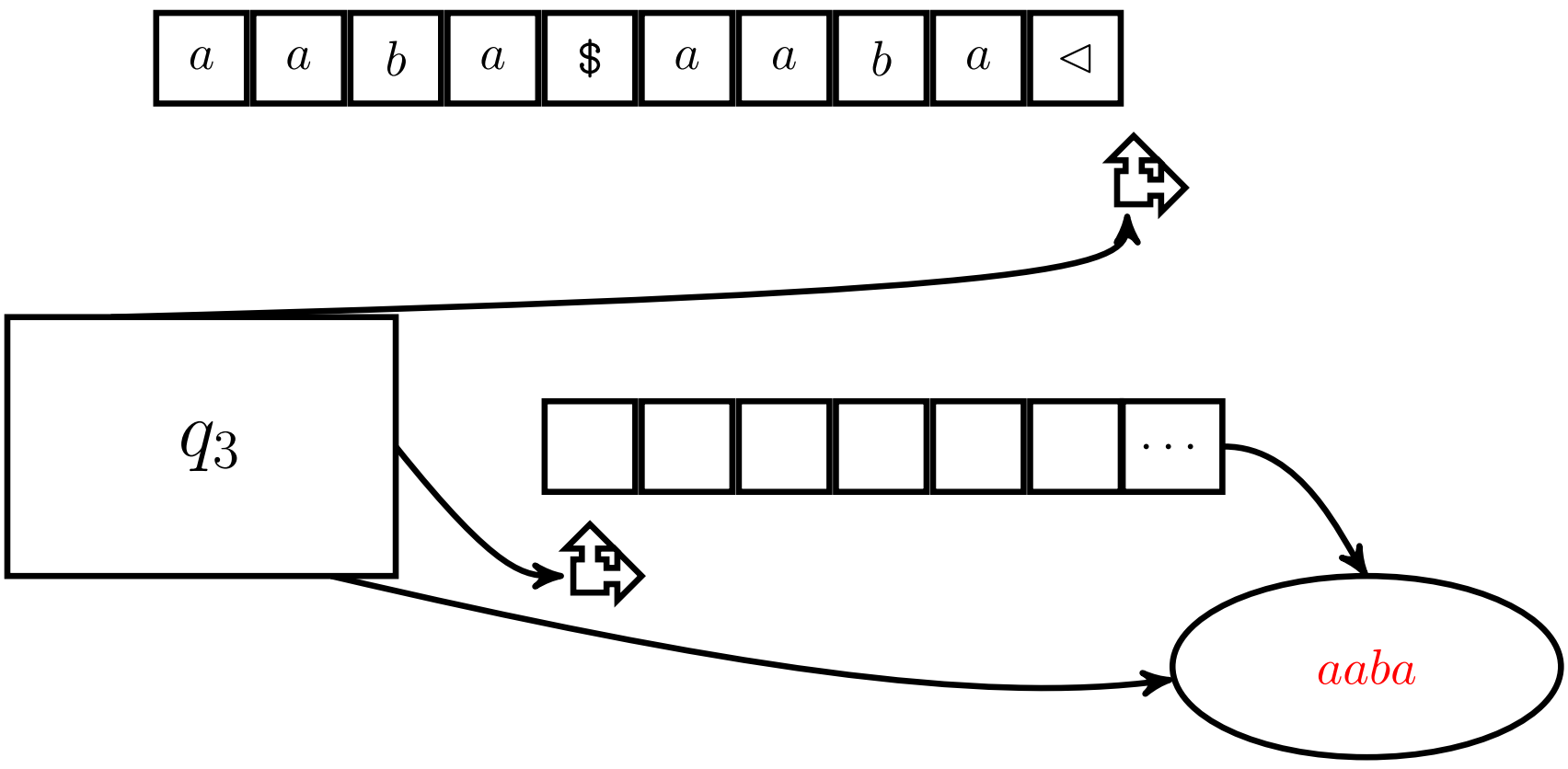

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The family of languages accepted by DSA is incomparable with the family of languages accepted by quasi-real-time queue automata.

- The non-regular language $\{ a^n \mid n \text{ is a Fibonacci number} \}$ is accepted by some quasi-real-time queue automaton.
- $L_2 = \{ a^n b^m \$_0 c^n \mid m, n \geq 1 \} \cup \{ a^n b^m \$_1 c^m \mid m, n \geq 1 \}$ is not accepted by any quasi-real-time queue automaton.

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- All sets L_{s_i,s_j} with s_i and s_j non-writing states that describe all words that can be written on the tape when the computation starts in state s_i with empty tape and ends in state s_j, and in between no other state performing an operation on the set is entered are infinite.

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- Second step: Construct an equivalent one-way multi-head finite automaton accepting L.

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Lemma

The family of languages accepted by DSA is not closed under union and intersection, but is closed under intersection with regular languages and under union with regular languages.

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Corollary

It is decidable whether the language accepted by a given deterministic set automaton is empty, finite, infinite, or equal to a given regular set.