Embedding finite and infinite words into overlapping tiles

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1. Overlapping tiles ?

Modeling tools for action refinement problems

A real world example

While flying across Russia

Flight SU 1406: the theory

$$\begin{array}{ccc} \downarrow & & \\ \mathit{Mos.} & \longrightarrow & \mathit{Ekat.} \\ & & \downarrow \end{array}$$

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Flight SU 1406: the refinement/implementation

$$\begin{array}{ccccc} \downarrow & & & & \\ Mos. & \longrightarrow & Perm & \longrightarrow & Ekat. \end{array}$$

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Flight SU 1406: the theory

$$\downarrow \qquad\qquad\qquad\qquad\qquad\qquad$$
$$Mos. \longrightarrow Ekat.$$
$$\qquad\qquad\qquad\qquad\qquad\qquad \downarrow$$

Flight SU 1406: the refinement/implementation

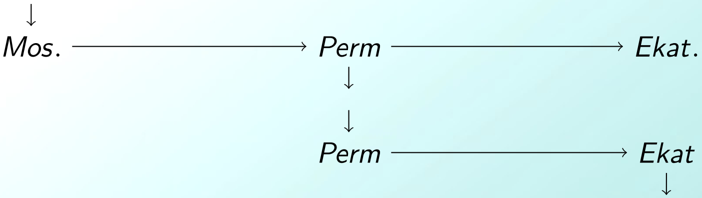
$$\downarrow \qquad\qquad\qquad\qquad\qquad\qquad$$
$$Mos. \longrightarrow Perm \longrightarrow Ekat.$$
$$\downarrow$$

A real world example

While flying across Russia

Flight SU 1406: the theory

Flight SU 1406: the refinement/implementation



Action refinement problem

The classical case

Refining two actions in sequence

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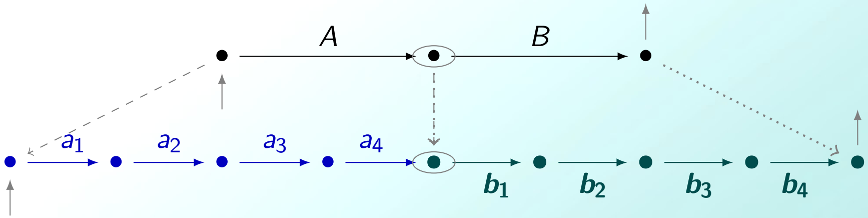
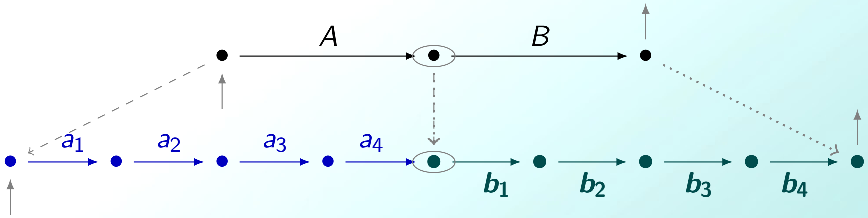
Refining two actions in sequence



Action refinement problem

The classical case

Refining two actions in sequence



Constraints : $a_1 a_2 a_3 a_4 \in L_A$ et $b_1 b_2 b_3 b_4 \in L_B$.

Action refinement problem

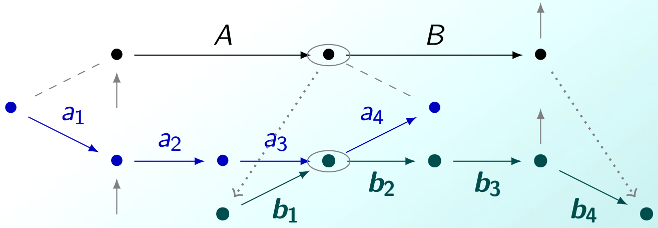
The extended case

Refining two actions in sequence. . . with overlaps

Action refinement problem

The extended case

Refining two actions in sequence. . . with overlaps



Our need

A well define, robust, versatile and expressive theory of

(languages of) overlapping structures !

2. Previous works

Mathematically well defined models

Available concepts and tools

Existing algebras of overlapping structures: (roof) tiles

- Right birooted words, Nivat and Perrot (1970), (used here)
- Birooted words, McAlister (1973) and Lawson (1998b),
- Birooted trees, Scheiblich (1972); Munn (1974),
- Birooted graphs, Stephen (1990),

with inverse semigroup theory (see Lawson (1998a)) in the background.

Existing language theory for overlapping structures

- Word languages and inverse semigroups, Margolis and Pin (1984); Margolis and Meakin (1993),
- Birooted tree languages and inverse semigroups, Silva (1996),

Ongoing development in Soft Eng.

Modeling with overlapping structures

- Music (or rhythmic) modeling: J. (2011) and followups, with variants of tiling semigroups Kellendonk and Lawson (2000),
- Distributed algorithm modeling, e.g. the dining philosophers by Dicky and J. (2013),
- Tiled temporal media programming, e.g. within Haskell by Hudak and J. (2014) at FARM.

Another example

Music driven waterworks

The music driven singing fountain on Ekaterinburg river which controller could well have been developed efficiently with the modeling and control tools we aim at providing in the next years.

A modeling example: beats in lyrics/poetry

Stressed syllabes in Dylan's Blowin' in the wind

The answer, my friend, is blowin' in the wind.

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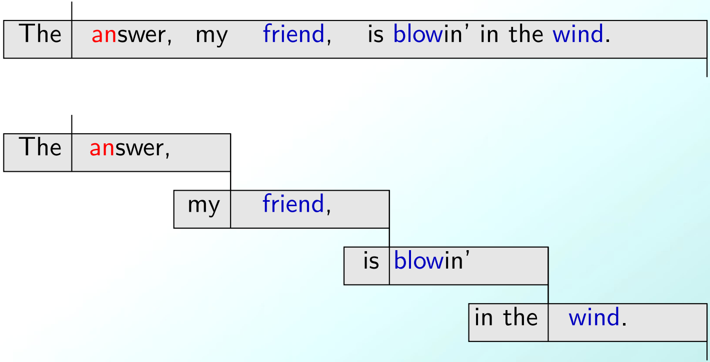

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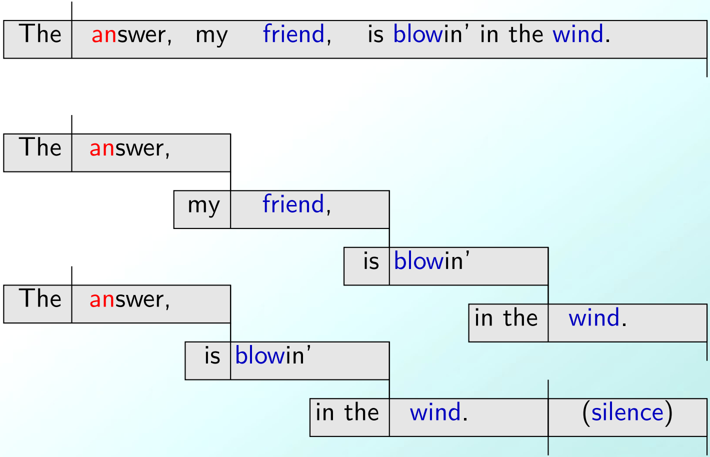


Ongoing development in Theoretical Comp. Sci.

Language theory for overlapping structures

- Birooted word languages and non deterministic automata, J. (2012) at MFCS and (2013) at CSR,
- Birooted tree languages and non deterministic automata, J. (2013) at ICALP,
- Birooted word languages and two way automata, following Pécuchet (1985), Dicky and J. (2012), unpublished,
- Tree walking automata and birooted trees, J. (2013), unpublished,
- Birooted graphs languages, J. (2014) at SOFSEM. . .

Our contribution at DLT

We study here finite and infinite birooted words and their languages for application in Tiled Programming where infinite tiles naturally arise thanks to lazy evaluation mechanism.

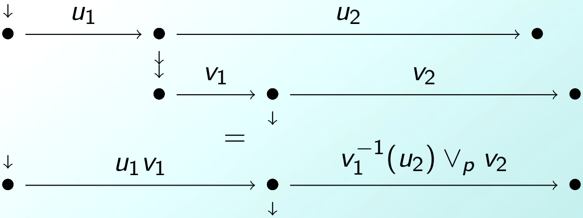
3. Finite and infinite tiles

Right finite and infinite tiles... to make it simpler

Right overlapping tiles (right birooted words)

Pairs of word $(u_1, u_2) \in A^* \times A^*$ drawn as a birooted words:

Birooted word products = synchronization + fusion



with (vertical) pattern matching conditions, 0 otherwise.

Right finite tiles

Remark

Right tiles form the polycyclic monoid $T^+(A)$ of Nivat and Perrot (1970) with unit $1 = (1, 1)$. This monoid can also be seen as a submonoid of the inverse monoid of birooted words $T(A)$ of McAlister (1973).

Definition (Natural order, Nambooripad (1980))

$x \leq y$ when $x = yz$ for some idempotent z.

Lemma

0 is minimum and $(u_1, u_2) \leq (v_1, v_2)$ if, and only if, $u_1 = v_1$ and $u_2 \geq_p v_2$.

Remark

Idempotent elements are subunits, i.e. $xx = x$ if and only if $x \leq 1$ in the natural order. We write $U(T^+(A))$ for the submonoid of subunits.

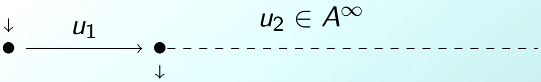
From finite to infinite tiles

Remember that a filter in $(T^+(A), \leq)$ is a non empty subset $F \subseteq T^+(A)$ such that:

- F is upward closed in the natural order, i.e. if $x \in F$ and $x \leq y$ then $y \in F$,
- F is downward directed, i.e. if $x, y \in F$ then $z \leq x$ and $z \leq y$ for some $z \in F$.

Lemma (Filters of $T^+(A)$)

Either of the form $F = \{(v_1, w) \in T^+(A) : w \leq_p v_2\}$ with $(v_1, v_2) \in A^* \times A^\infty$, or of the form $F = T^+(a)$.



Lemma (Filter completion)

The set $T^\infty(A) \sim 0 + A^* \times A^\infty$ of filters over $T^+(A)$ equipped with point-wise product is a monoid: the monoid of finite and infinite right overlapping tiles.

Embedding finite and infinite words into tiles

Theorem (Dicky and J. at DLT)

The pair of mappings

$$\varphi = \langle \varphi_f, \varphi_\omega \rangle : \langle A^+, A^\omega \rangle \to T^\infty(A)$$

defined by

$$\varphi_f(u_1) = (u_1, 1) \text{ and } \varphi_\omega(u_2) = (1, u_2)$$

induces an ω-semigroup embedding of the free ω-semigroup $\langle A^+, A^\omega \rangle$ into the ω-semigroup $\langle T^+(A), U(T^\infty(A)) \rangle$

- *finite product: $x \cdot y = xy$ (product in $T^\infty(A)$),*
- *mixed product: $x * y = (x \cdot y)^R$ with $0^R = 0$ and $(u_1, u_2)^R = (1, u_1 u_2)$,*
- *infinite product: $\prod (x_i)_{i \in \omega} = \bigwedge_k (\prod_{i \leq k} x_i)^R$.*

4. Finite and infinite tile languages

Allowing to embed ω-semigroup into (some notion of) ω-adequately ordered monoids in a mathematically robust way.

Language theoretic tools: the finite case

Remark

In (quasi-) inverse semigroups, recognizability by morphism collapses (see e.g. J. (2012) at SOFSEM).

Word morphism lifting

Given $\varphi : A^* \to M$ with monoid M define

$$\begin{array}{rcl} \psi : T^+(A) & \to & M \times R_{\mathcal{I}}(M) \\ (u_1, u_2) & \mapsto & (\varphi(u_1), \varphi(u_2 A^*)) \end{array}$$

Lemma (See J. (2012) at MFCS)

The structure $M \times R_{\mathcal{I}}(M)$ equipped with the product

$$(x, X) \cdot (y, Y) = (xy, y^{-1}(X) \cap Y)$$

is an adequately ordered monoid with ψ an adequate premorphism.

Language theoretic tools: the finite and infinite cases

Word morphism lifting extended to infinite tiles

Given $\varphi : A^* \to M$ with monoid M. Given $\overline{M} = (M, M_\omega)$ the ω-completion of M and $\varphi_\omega : A^\omega \to M_\omega$ given by ω-semigroup theory (see Perrin and Pin (2004)). Then, define

$$\begin{array}{rcl} \psi : T^\infty(A) & \to & M \times \mathcal{P}(M_\omega) \\ (u_1, u_2) & \mapsto & (\varphi(u_1), \varphi_\omega(u_2 A^\omega)) \end{array}$$

Lemma (Dicky and J. (2014) at DLT)

The structure $M \times \mathcal{P}(M_\omega)$ equipped with the product

$$(x, X) \cdot (y, Y) = (xy, y^{-1}(X) \cap Y)$$

is an adequately ordered ω-monoid with ψ an adequate premorphism.

ω-Quasi-recognizability

Theorem (Dicky and J. (2014) at DLT)

Let $L \subseteq T^{+\infty}(A)$, then the following property are equivalent:

(i) *L is a finite boolean combination of upward closed (in the natural order) MSO definable languages,*

(ii) *there exists a finite ω-adequately ordered monoid M and an adequate premorphism*

$$\psi : T^{\infty}(A) \to M$$

such that $L = \psi^{-1}(\psi(L))$, that is, M recognizes L via the premorphism ψ.

Embedding ω-semigroups into ω-adequate monoids

Theorem (Dicky and J. (2014) at DLT)

Let $S = \langle S_f, S_\omega \rangle$ be an ω-semigroup. Given $M(S) = S_f \times \mathcal{P}(S_\omega)$ with same product as above. Then, the pair of mappings

$$\langle \varphi_f, \varphi_\omega \rangle : S \to M$$

defined, for every $x_1 \in S_f$ and $x_2 \in S_\omega$, by

$$\varphi_f(x_1) = (x_1, 1) \text{ and } \varphi_\omega(x_2) = (1, \{x_2\})$$

induces an ω-monoid morphisms from S into $\langle M(S), U(M(S)) \rangle$ equipped with adequate finite, mixed and infinite product.

5. Conclusion

As a temporary end.

What has been proposed

- Embedding two-sorted ω-monoids into one sorted ordered ω-monoids is easy. . . once good definitions have been found.
- This allows to lift the algebraic language theory of languages of ω-words to a quasi-algebraic langage theory of languages of ω-tiles.

Question

- Recognizability by adequate premorphisms extends to finite birooted tree languages, J. (2013), ICALP !
- Here, we have extended recognizability by adequate premorphism to finite and infinite birooted words !
- Could these two approches be combined to adress the emerging (and DIFFICULT) algebraic theory of languages of infinite trees, Blumensath (2011) ?

And that's all folks !
Thank for your attention !

A. Blumensath. Recognisability for algebras of infinite trees. *Theoretical Comp. Science*, 412(29):3463–3486, 2011.

A. Dicky and D. Janin. Two-way automata and regular languages of overlapping tiles. Research report RR-1463-12, LaBRI, Université de Bordeaux, 2013a. URL `http://hal.archives-ouvertes.fr/hal-00717572`.

A. Dicky and D. Janin. Modélisation algébrique du diner des philosophes. In *Modélisation des Systèmes Réactifs (MSR), in Journal Européen des Systèmes Automatisés (JESA Volume 47 - no 1-2-3/2013)*, 2013b. doi: http://dx.doi.org/10.3166/jesa.47.29-43. URL `http://hal.archives-ouvertes.fr/hal-00856678`.

A. Dicky and D. Janin. Embedding finite and infinite words into overlapping tiles. In *Developments in Language Theory (DLT)*, volume 8633 of *LNCS*, pages 339–347, Ekaterinburg, Russia, 2014. Springer. URL `http://hal.archives-ouvertes.fr/hal-00910002`.

P. Hudak and D. Janin. Tiled polymorphic temporal media. In *ACM Workshop on Functional Art, Music, Modeling and Design*

(FARM), page (in print), Gothenburg, Sweden, 2014. ACM Press. URL http://hal.archives-ouvertes.fr/hal-00955113.

D. Janin. Vers une modélisation combinatoire des structures rythmiques simples de la musique. *Revue Francophone d'Informatique Musicale (RFIM)*, 2, 2012a. URL http://hal.archives-ouvertes.fr/hal-00608295.

D. Janin. Quasi-recognizable vs MSO definable languages of one-dimensional overlapping tiles. In *Mathematical Found. of Comp. Science (MFCS)*, volume 7464 of *LNCS*, pages 516–528, 2012b. doi: http://dx.doi.org/10.1007/978-3-642-32589-2_46. URL http://hal.archives-ouvertes.fr/hal-00671917.

D. Janin. On languages of one-dimensional overlapping tiles. In *Int. Conf. on Current Trends in Theo. and Prac. of Comp. Science (SOFSEM)*, volume 7741 of *LNCS*, pages 244–256. Springer, 2013a. doi: http://dx.doi.org/10.1007/978-3-642-35843-2_22. URL http://hal.archives-ouvertes.fr/hal-00659202.

D. Janin. Walking automata in the free inverse monoid. Research report RR-1464-12, LaBRI, Université de Bordeaux, 2013b. URL http://hal.archives-ouvertes.fr/hal-00738793.

D. Janin. Overlaping tile automata. In *8th International Computer Science Symposium in Russia (CSR)*, volume 7913 of *LNCS*, pages 431–443. Springer, 2013c. doi: http://dx.doi.org/10.1007/978-3-642-38536-0_37. URL http://hal.archives-ouvertes.fr/hal-00766261.

D. Janin. Algebras, automata and logic for languages of labeled birooted trees. In *Int. Col. on Aut., Lang. and Programming (ICALP)*, volume 7966 of *LNCS*, pages 318–329. Springer, 2013d. doi: http://dx.doi.org/10.1007/978-3-642-39212-2_29. URL http://hal.archives-ouvertes.fr/hal-00784898.

D. Janin. Towards a higher dimensional string theory for the modeling of computerized systems. In *Int. Conf. on Current Trends in Theo. and Prac. of Comp. Science (SOFSEM)*, volume 8327 of *LNCS*, pages 7–20. Springer, 2014. doi: http://dx.doi.org/10.1007/978-3-319-04298-5_2. URL http://hal.archives-ouvertes.fr/hal-00879463.

J. Kellendonk and M. V. Lawson. Tiling semigroups. *Journal of Algebra*, 224(1):140 – 150, 2000.

M. V. Lawson. *Inverse Semigroups : The theory of partial symmetries*. World Scientific, 1998a.

M. V. Lawson. McAlister semigroups. *Journal of Algebra*, 202(1): 276 – 294, 1998b.

S. W. Margolis and J. C. Meakin. Inverse monoids, trees and context-free languages. *Trans. Amer. Math. Soc.*, 335:259–276, 1993.

S. W. Margolis and J.-E. Pin. Languages and inverse semigroups. In *Int. Col. on Aut., Lang. and Programming (ICALP)*, volume 172 of *LNCS*, pages 337–346. Springer, 1984.

D.B. McAlister. Inverse semigroups which are separated over a subsemigroups. *Trans. Amer. Math. Soc.*, 182:85–117, 1973.

W. D. Munn. Free inverse semigroups. *Proceeedings of the London Mathematical Society*, 29(3):385–404, 1974.

K. S. S. Nambooripad. The natural partial order on a regular semigroup. *Proc. Edinburgh Math. Soc.*, 23:249–260, 1980.

M. Nivat and J.-F. Perrot. Une généralisation du monoïde bicyclique. *Comptes Rendus de l'Académie des Sciences de Paris*, 271:824–827, 1970.

J.-P. Pécuchet. Automates boustrophedon, semi-groupe de Birget et monoide inversif libre. *ITA*, 19(1):71–100, 1985.

D. Perrin and J.-E. Pin. *Infinite Words: Automata, Semigroups, Logic and Games*, volume 141 of *Pure and Applied Mathematics*. Elsevier, 2004.

H. E. Scheiblich. Free inverse semigroups. *Semigroup Forum*, 4: 351–359, 1972.

P. V. Silva. On free inverse monoid languages. *ITA*, 30(4): 349–378, 1996.

J.B. Stephen. Presentations of inverse monoids. *Journal of Pure and Applied Algebra*, 63:81–112, 1990.